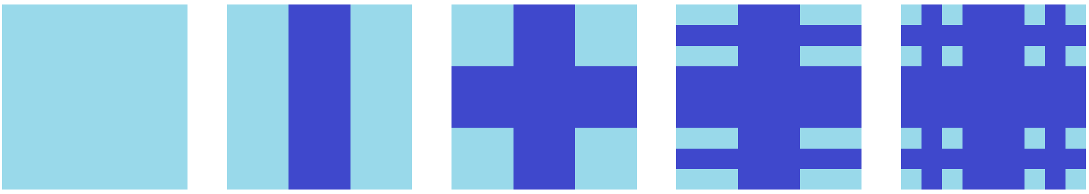
Extension – day 1

Light Blue - Dark Blue

Age 7 to 11
Challenge Level ★★

Look at each of these five squares.



What is the pattern?

How much of the second square is light blue? Can you write this as a fraction?

For each of the five squares, write the area of the square that is light blue as a fraction.

Can you work out what the next two diagrams would look like? What fraction of these squares will be light blue?

Day 1 – extension solution

Light Blue - Dark Blue

Age 7 to 11
Challenge Level ★★

Francesca investigated this problem. She imagined that each time the big square was split up into little blocks that looked like the light blue ones. Then she counted how many light blue ones there were, and how many overall. This is what she got :

$1, \frac{2}{3}, \frac{4}{9}, \frac{8}{27}, \frac{16}{81}, \ldots$

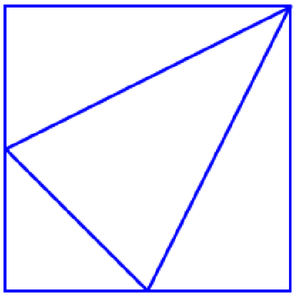
She noticed that the number on top got multiplied by 2 each time, and the number on the bottom got multiplied by 3 each time.
Some of our more advanced readers might know that we could write this as

$\frac{2^n}{3^n}.$

Francesca also noticed that the amount of light blue got smaller and smaller each time. She thinks that if we could do this forever, in the end the whole square would be dark blue.

Day 2 Extension

Fraction Fascination

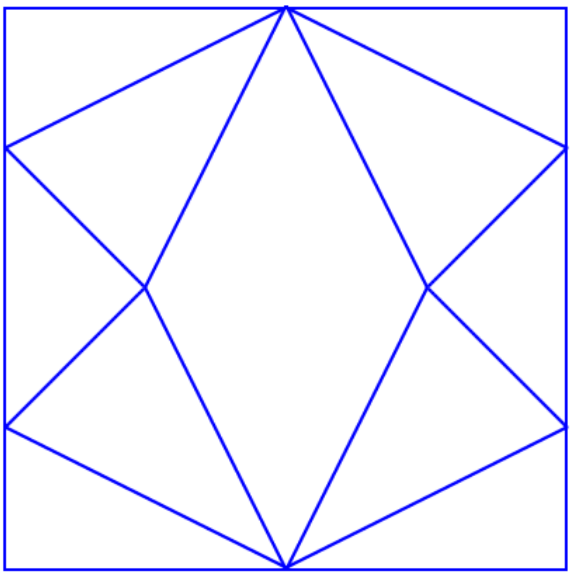


I drew this picture by drawing a line from the top right corner of a square to the midpoint of each of the opposite sides. Then I joined these two midpoints with another line.

Can you see four triangles in the square?

What fraction of the area of the square is each of these triangles?

Then I drew another picture:



How is this made using the first square?

What is the shape that has been created in the middle of this larger square?

What fraction of the total area of the large square does this shape take up?

Day 2 extension - solution

Fraction Fascination

Age 7 to 11
Challenge Level
We had some good solutions come in for this challenge.

Sophie and Emma, Keenan and Dylan from Crossacres Primary School wrote:

The smaller square: We traced the shape on tracing paper. Then we cut the triangle shapes out. We found out that the two big triangles fitted to make half (we had to flip one over). Because there were two of them, we halved a half which makes 14.

Then we saw that the small triangle was half of a quarter which is 18.

Then we turned all of the fractions into eighths and added them together and got 58.

This means that the last triangle (the big one) is equal to 58 because 58 + 58 = 1 whole.

and

You will need scissors and more than one copy of the shapes. First we cut out the four triangles that make up the small square. Then we took the two right-angled triangles so that they made an oblong. We put the oblong on top of another copy of the square.The two oblongs covered a half of the square, that makes them a quarter each. This means that the large triangle and small triangle together make up a half too. We then made a square using two of the small triangles. When we put this square on the original square we could see that it equals a quarter. That means that the small triangle is equal to one eighth because that is half of a quarter. The large triangle is equal to three eighths. Work this out by adding one quarter + one quarter + one eighth = five eighths and then to make 1 whole it is three eighths (the largest triangle).

When we looked at the second big square straight away we could see that it was made up of four of the smaller squares. That means that each of the fractions we first worked out would be four times smaller. We separated the large square into the four smaller squares and wrote down what each of the triangles was now worth. To work out the size of the rhombus we worked out
116 + 116 + 116 + 116 = 416.

Then we simplified the fraction to 14.

Then we checked our answer by using four cut out triangles we placed them on the square and they covered one quarter of the shape.

Esme from St John's CEP School sent in the following:

For the first picture I drew two lines to make quarters on the picture. In the top half one triangle is half of the half (to make the fractions easy to understand I changed them into eighths) which is two eighths and as the top triangle is the same as the left hand side triangle you know that the left hand side triangle is also two eighths.
In the bottom right hand corner the triangle is half the quarter so the triangle is one eighth.
Then if you add up all the eighths you get five eighths that means that the remaining centre triangle is three eighths.

For the second picture I split it into sixteenths but I calculated it in to thirty-twoths so I could work out the answer more easily.
In the top right hand corner triangle it was half of two sixteenths which (in thirty-twoths) is two thirty-twoths. That is the same as each corner and the quarters of the big middle shape which means overall the middle shape is eight thirty-twoths.
In the centre of the right and left hand side each side's two sixteenths is one sixteenth of the two sixteenths. That means that each side triangle is two thirty-twoths.
Then if you add up the triangles you have calculated so far you get twenty thirty-twoths so the remaining shapes have twelve thirty-twoths all together so each of the remaining triangles is three thirty-twoths.

Lydia also from St. John's wrote:

I did this by looking at the bottom line and realising that the central triangle touched halfway, so I drew a vertical line from this point upwards. This then shows that the triangle on the right is half of the rectangle you have just created, therefore the triangle is a quarter of the entire square.
On the left hand side the central triangle's point touches halfway too and I drew the line horizontally across; now the square is in quarters. The top two quarters are split in half by a second triangle which makes that triangle a quarter and together with the triangle on the right a half. Half of one of the smaller squares (bottom left) is filled with a tiny right-angled triangle, and half a quarter is an eighth so the small triangle is one-eighth of the entire square. So now you have one-eighth and a half which makes five eighths. 8 minus 5 equals 3 so the central triangle is three as that is the remaining value!

So this is the answer:

Bottom left triangle = 18; Top triangle = 28; Triangle to the right = 28;   Central triangle = 38

Here is a response from Esther and Tracy from Withernsea Primary School:

First we drew a vertical and horizontal line in order to split the square into quarters. We then looked at the triangle on the left of the square and

decided that it was half of the left half of the square. This meant that it was a quarter of the whole square.
The triangle at the top of the square was also a quarter.
The smallest triangle was half the size of these two. Half of a quarter is an eighth.
So, the three triangles were a quarter, a quarter and an eighth.
To calculate the last triangle we turned the denominators into eighths. So, they became 28, 28 and 18. This meant that the central triangle must have been 38 as: 28 + 28 + 18 + 38 = 88.

For the next problem we again divided the square into quarters. Looking at the top left hand square, we decided that the part of the central shape was a quarter of this square. This means that this must be one sixteenth of the whole shape. As there is are four of these triangles, they must take up four sixteenths of the whole square. So the central shape is four sixteenths or one quarter of the whole square.

Sakura, Ema and Wictoria at the VIenna International School in Austria we had sent in the following;

We first labelled the four triangles in size order A, B, C, D. Then we cut the triangles up and noticed that triangles B + C were equal sizes and together measured half the square. Therefore B was 14 and C was 14 of the total area.

Next we split the original diagram into four quadrants. We identified that triangle D was half of a quadrant and therefore equated to 18 of the total area.

We then converted all of our fractions to eighths. We had 28, 28 + 18 = 58. Therefore we knew triangle A would be 38 as the whole would add up to 88.
We checked our answer by cutting the triangles further.

Zach sent in an excellent account of his method of finding solutions. He began by saying:

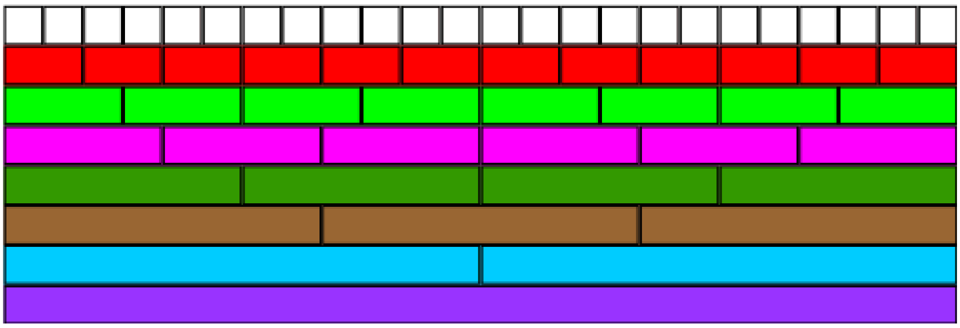
Start by labelling the triangles. Note that Triangles A, C & D are right angled, whilst Triangle B is isosceles. Triangle A = Triangle C

To read Zach's full solution, see one of the following:Fraction Fascination - Zach.docx or PDF Fraction Fascination - Zach .pdf

Day 3 extension

Fractional Wall

Age 7 to 11
Challenge Level ★



Using the image above, how many different ways can you find of writing $\frac{1}{2}$?

From the picture, what equivalent fractions for $\frac{1}{3}$ can you find?

Again, using the image of the fraction wall, how else could you write $\frac{3}{4}$?

What other fractions do you know that are the same as $\frac{1}{2}$?

Find some other fractions which are equivalent to $\frac{3}{4}$.

Can you find any "rules" for working out equivalent fractions?

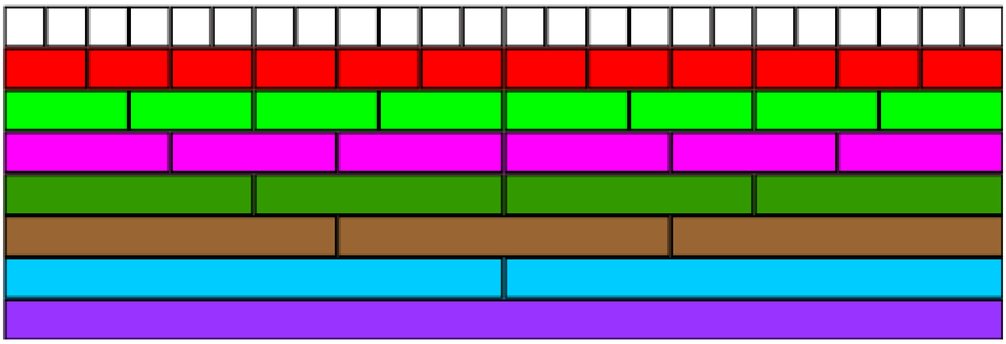
You might find it helpful to print off [a picture of the fraction wall].

Day 3 Extension - solution

Fractional Wall

Age 7 to 11
Challenge Level

Thanks to the many of you who submitted a solution to this problem. Congratulations to Josie, Dominic and George from St Nicholas CE Junior School, Newbury and also Abigail from Histon Junior School and Charlie from Beckley C of E who all sent in clearly explained solutions. The solution given below was sent in by Cong:



Using the image above, I can find 12 as:
1 blue (12)
2 dark greens (24)
3 pinks (36)
4 light greens (48)
6 reds (612)
12 whites (1224)

So I can also say that

$$12=24=36=48=612=1224$$

From the picture, I can find 13 as:
1 brown (13)
2 pinks (26)
4 reds (412)
8 whites (824)

So I can also say that

$$13=26=412=824$$

Again, using the image of the fraction wall, I can find 34 as:

3 dark greens (34)
6 light greens (68)
9 reds (912)
18 whites (1824)

So again I can say that

$$34=68=912=1824$$

The rule for working out equivalent fractions is to multiply the numerator and the denominator with the same whole number.

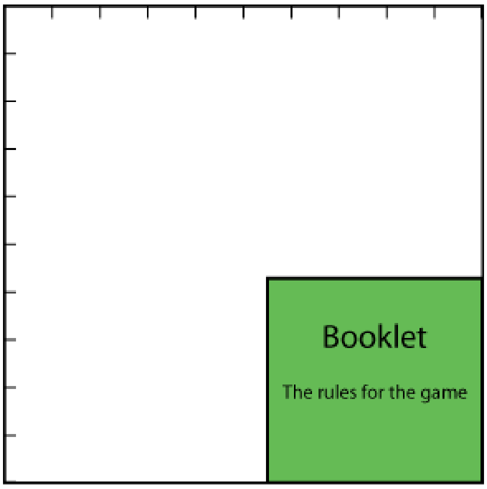
Day 4 Extension

Fractions in a Box

Age 7 to 11
Challenge Level

We have a game which has a number of discs in seven different colours. These are kept in a flat square box with a square hole for each disc. There are 10 holes in each row and 10 in each column. So, there would be 100 discs altogether, except that there is a square booklet which is kept in a corner of the box in place of some of the holes.

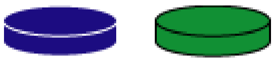
We haven't drawn a grid to show all the holes because that would give the answer away!



There is a different number of discs of each of the seven colours.

Half (12) of the discs are red, 14 are black and 112 are blue.

One complete row (of 10 holes) of the box is filled with all the blue and green discs.



One of the shortened rows (that is where the booklet is) is exactly filled with all the orange discs.

Two of the shortened rows are filled with some of the red discs and the rest of the red discs exactly fill a number of complete rows (of 10) in the box.

There is just one white disc and all the rest are yellow.

How many discs are there altogether?
What fraction of them are orange?
What fraction are green? Yellow? White?

Day 4 Extension – Solution

Fractions in a Box

Age 7 to 11
Challenge Level

This was a tricky problem. Well done to those of you who had a go. We had some very clearly explained answers. The key was to work out the size of the booklet first.

Rachel, Ol, Jack and Alex from Moretonhampstead Primary said:

First we worked out how many squares the booklet is. The number has to be a square number and has to be even and 14 of that number has to be even again. The only number possible for that is 16 (four squared).
Then we took 16 (that was how big the booklet was) from a 100 which is 84 (there are 84 squares to play the game with). With 84 we can answer the first question - how many discs are there altogether? (84).
After that we worked out how many discs there would be for the colours. We worked out there would be 42 discs (12 of 84), 21 black discs (14 of 84) and 7 blue (112 of 84).
We put that on the grid as it says on the sheet. Next we used the last full column for blue and green. We know that there are 7 blues (because of what we worked out earlier) which means there are 3 green discs (7+3=10 (which is how many in a column)).
There were five squares left. It says that there is 1 white square then the leftovers are yellow so we had 4 yellows and 1 white disc.
Now we hade completed the grid we could answer question 2 and 3. For question 2 we counted up all the orange squares (6) and the fraction is 684 but we had to simplify it to 114 (the answer).
Lastly we did question 3. The fraction of green is 384 and simplified for the actual answer is 128.
The fraction of yellow is 484 and simplified for the answer is 121.
There is only one white square so the answer is 184, for white.

Hamish, Rory, Sarah, Jesse and Samuel from Rutherglen Primary also reasoned very clearly and they sent us a picture of the full box which they modelled using cubes:

Sophie and Claire from The Downes School wrote:

1×1 didn't work because it said that **two** shortened rows have red discs.
2×2 didn't work because you need **two** shortened rows of red and **one** of orange.
3×3 didn't work because the total number of discs would be odd and you couldn't halve it. This means all odd numbers didn't work.
4×4 did work because you had the right amount of shortened rows.
6×6 didn't work because you can't divide 64 by 12.
8×8 didn't work because you need **six** whole rows.

Emma, Abi, Matthew B and Yuji from Moorfield Junior School; Keshinie and Sharon at Kilvington GGS Victoria, Australia; Gideon from Newberries Primary School and Hannah, Georgia, Patrick; Hana from Bali International School and Matthew from Brighton College Prep School realised that the number left after taking away the booklet must be a multiple of 12. Keshinie and Sharon describe how they continued from there:

So that made it 84.
Half of the disks are red so that made the amount of red 42.

Then it said that a quarter is black so that made it 21.
Then it said that one twelfth is blue so that made it 7.

Then it said that one complete row was filled with all of blue and green and the remainder of 10 if you take away 7 made it 3 green.

Then it said that one of the shortened rows is exactly filled with all the orange disks so that makes it 6.

Then it said that there was only one white disk.
Then we added all the numbers together making 80 disks so there was a remainder of 4 which had to be yellow.

We divided the 84 disks by the 6 orange ones that made it 14. So the fraction of orange had to be 1 out of 14 (114).
We divided the 84 disks by the 3 green disks making the answer 28. So the fraction of green had to be 128.
We already knew that the fraction of white disk was 184.
We divided the 84 disks by the 4 yellow ones making it 21 so the fraction of yellow had to be 121.

James from the Charter School explained very well how he went about the problem:

Each of the sides is 10 units and I called each of the sides of the booklet x. this means that the equation for finding the number of discs was $N=100-x2$. (N being the number of remaining discs).
The amount of Blue discs was $N12$ meaning that N was a multiple of 12.
So I then collected all of the multiples of 12: 12, 24, 36, 48, 60, 72, 84, 96. I then eliminated those that did not fit the earlier equation because there was not a square number that fitted. This left: 36, 84, 96.
I eliminated 96 because x had to be more that three for there was one complete row of orange discs and two of red discs.
This left: 36, 84. From this I deduced that x had to be 4 or 8. This means that the amount of red disks had to end in a 2 or a 4, because there are two incomplete rows of red these either have to be a length of 6 or 2.
Since the amount of red discs is half of N I halved both my possible N's which came up with 18 and 42. This means that the amount of reds was 42 and N was 84.
This means that x is 4 and that the amount of orange discs was 6 meaning the fraction is 114.
The amount of blue was $N12$ which was 7.
This means that the amount of green discs was 3 because blues + greens = 10. That means the fraction was 128.
The amount of white was 1 meaning the fraction was 184.
Finally the rest were yellow.

Red was 42. Blacks was $N4$ which was 21. Blue was 7. Orange was 6. Green was 3. White was 1.
If you take all those away from 84 you end up with 4. That is the amount of yellows. This means the fraction is 121.

Day 5 Extension

Chocolate

This challenge is about chocolate. You have to imagine (if necessary!) that everyone involved in this challenge enjoys chocolate and wants to have as much as possible.

There's a room in your school that has three tables in it with plenty of space for chairs to go round. Table 1 has one block of chocolate on it, table 2 has two blocks of chocolate on it and, guess what, table 3 has three blocks of chocolate on it.

Now ... outside the room is a class of children. Thirty of them all lined up ready to go in and eat the chocolate. These children are allowed to come in one at a time and can enter when the person in front of them has sat down. When a child enters the room they ask themself this question:

"If the chocolate on the table I sit at is to be shared out equally when I sit down, which would be the best table to sit at?"

However, the chocolate is not shared out until all the children are in the room so as each one enters they have to ask themselves the same question.

It is fairly easy for the first few children to decide where to sit, but the question gets harder to answer, e.g.

It maybe that when child 9 comes into the room they see:

- 2 people at table 1
- 3 people at table 2
- 3 people at table 3

So, child 9 might think:

"If I go to:

- *table 1 there will be 3 people altogether, so one block of chocolate would be shared among three and I'll get one third.*
- *table 2 there will be 4 people altogether, so two blocks of chocolate would be shared among four and I'll get one half.*
- *table 3, there will be 4 people altogether, so three blocks of chocolate would be shared among four and I'll get three quarters.*

Three quarters is the biggest share, so I'll go to table 3."

Go ahead and find out how much each child receives as they go to the "best table for them". As you write, draw and suggest ideas, try to keep a note of the different ideas, even if you get rid of some along the way.

THEN when a number of you have done this, talk to each other about what you have done, for example:

A. Compare different methods and say which you think was best.

B. Explain why it was the best.

C. If you were to do another similar challenge, how would you go about it?

Chocolate

Age 7 to 14
Challenge Level

There were a few complete solutions sent in and many who showed us what the final result at the tables would be, i.e. 5 at the 1 table, 10 at the 2 table and 15 at the 3 table.

The full solution showing fractions at each stage were received from Adriel, Emily and Aswaath from the Garden International School in Maylasia. Also Daniel at Staplehurst School and Megan at Twyford School. Here is Emily's contribution.

So after child 9 has sat down, there are now:

2 people at table 1

3 people at table 2

4 people at table 3

This is a list of which table child $10-30$ would go to:
The reason I listed down which table they would go to is to see whether there was a pattern

Child 10 would go to table 3 and receive ¾ of the chocolate

Child 11 would go to table 3 and receive ½ of the chocolate

Child 12 would go to table 2 and receive ½ of the chocolate

Child 13 would go to table 3 and receive $3/7$ of the chocolate

Child 14 would go to table 2 and receive $2/5$ of the chocolate

Child 15 would go to table 3 and receive $3/8$ of the chocolate

Child 16 would go to table 1 and receive $1/3$ of the chocolate

Child 17 would go to table 3 and receive $1/3$ of the chocolate

Child 18 would go to table 2 and receive $1/3$ of the chocolate

Child 19 would go to table 3 and receive $3/10$ of the chocolate

Child 20 would go to table 2 and receive $2/7$ of the chocolate

Child 21 would go to table 3 and receive $3/11$ of the chocolate

Child 22 would go to table 1 and receive ¼ of the chocolate

Child 23 would go to table 2 and receive ¼ of the chocolate

Child 24 would go to table 3 and receive ¼ of the chocolate

Child 25 would go to table 3 and receive $3/13$ of the chocolate

Child 26 would go to table 2 and receive $2/9$ of the chocolate

Child 27 would go to table 3 and receive $3/14$ of the chocolate

Child 28 would go to table 1 and receive $1/5$ of the chocolate

Child 29 would go to table 2 and receive $1/5$ of the chocolate

Child 30 would go to table 3 and receive $1/5$ of the chocolate

So now there are:

5 people in table 1

10 people in table 2

15 people in table 3

I found out a pattern from child 10 to child 30. When it was child 16's turn to decide which was the best place to sit, child 16 could just choose to randomly sit on any table because he'll still get $1/3$ of the chocolate in any of the tables. Same with child 22 and 28. Child 22 would either ways get ¼ and child $281/5$. The numbers also increase by $6.16,22,28$.

I noticed that sometimes there were tables that shares the same amount of chocolate so you could choose randomly between 2 or all of the tables which was what I did when I tried finding out how many chocolates each child receives as they go to find the best table for them

For example: Child 16 gets to choose between any of the 3 tables because he'll still get $1/3$ of the chocolate in any of the tables