

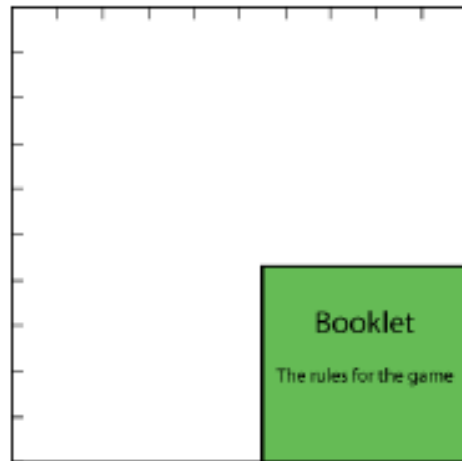
Fractions in a Box

Age 7 to 11

Challenge Level

We have a game which has a number of discs in seven different colours. These are kept in a flat square box with a square hole for each disc. There are 10 holes in each row and 10 in each column. So, there would be 100 discs altogether, except that there is a square booklet which is kept in a corner of the box in place of some of the holes.

We haven't drawn a grid to show all the holes because that would give the answer away!



There is a different number of discs of each of the seven colours.

Half (12) of the discs are red, 14 are black and 112 are blue.



One complete row (of 10 holes) of the box is filled with all the blue and green discs.



One of the shortened rows (that is where the booklet is) is exactly filled with all the orange discs.



Two of the shortened rows are filled with some of the red discs and the rest of the red discs exactly fill a number of complete rows (of 10) in the box.

There is just one white disc and all the rest are yellow.



How many discs are there altogether?

What fraction of them are orange?

What fraction are green? Yellow? White?

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This was a tricky problem. Well done to those of you who had a go. We had some very clearly explained answers. The key was to work out the size of the booklet first.

Rachel, Ol, Jack and Alex from Moretonhampstead Primary said:

First we worked out how many squares the booklet is. The number has to be a square number and has to be even and 14 of that number has to be even again. The only number possible for that is 16 (four squared).

Then we took 16 (that was how big the booklet was) from a 100 which is 84 (there are 84 squares to play the game with). With 84 we can answer the first question - how many discs are there altogether? (84).

After that we worked out how many discs there would be for the colours.

We worked out there would be 42 discs (12 of 84), 21 black discs (14 of 84) and 7 blue (112 of 84).

We put that on the grid as it says on the sheet. Next we used the last full column for blue and green. We know that there are 7 blues (because of what we worked out earlier) which means there are 3 green discs (7+3=10 (which is how many in a column)).

There were five squares left. It says that there is 1 white square then the leftovers are yellow so we had 4 yellows and 1 white disc.

Now we had completed the grid we could answer question 2 and 3. For question 2 we counted up all the orange squares (6) and the fraction is $\frac{6}{84}$ but we had to simplify it to $\frac{1}{14}$ (the answer).

Lastly we did question 3. The fraction of green is $\frac{3}{84}$ and simplified for the actual answer is $\frac{1}{28}$.

The fraction of yellow is $\frac{4}{84}$ and simplified for the answer is $\frac{1}{21}$.

There is only one white square so the answer is $\frac{1}{84}$, for white.

Hamish, Rory, Sarah, Jesse and Samuel from Rutherglen Primary also reasoned very clearly and they sent us a picture of the full box which they modelled using cubes:



Sophie and Claire from The Downes School wrote:

1×1 didn't work because it said that **two** shortened rows have red discs.
 2×2 didn't work because you need **two** shortened rows of red and **one** of orange.
 3×3 didn't work because the total number of discs would be odd and you couldn't halve it. This means all odd numbers didn't work.
 4×4 did work because you had the right amount of shortened rows.
 6×6 didn't work because you can't divide 64 by 12.
 8×8 didn't work because you need **six** whole rows.

Emma, Abi, Matthew B and Yuji from Moorfield Junior School; Keshinie and Sharon at Kilvington GGS Victoria, Australia; Gideon from Newberries Primary School and Hannah, Georgia, Patrick; Hana from Bali International School and Matthew from Brighton College Prep School realised that the number left after taking away the booklet must be a multiple of 12. Keshinie and Sharon describe how they continued from there:

So that made it 84.
Half of the disks are red so that made the amount of red 42.

Then it said that a quarter is black so that made it 21.

Then it said that one twelfth is blue so that made it 7.

Then it said that one complete row was filled with all of blue and green and the remainder of 10 if you take away 7 made it 3 green.

Then it said that one of the shortened rows is exactly filled with all the orange disks so that makes it 6.

Then it said that there was only one white disk.

Then we added all the numbers together making 80 disks so there was a remainder of 4 which had to be yellow.

We divided the 84 disks by the 6 orange ones that made it 14. So the fraction of orange had to be 1 out of 14 (1/14).

We divided the 84 disks by the 3 green disks making the answer 28. So the fraction of green had to be 1/28.

We already knew that the fraction of white disk was 1/84.

We divided the 84 disks by the 4 yellow ones making it 21 so the fraction of yellow had to be 1/21.

James from the Charter School explained very well how he went about the problem:

Each of the sides is 10 units and I called each of the sides of the booklet x . this means that the equation for finding the number of discs was $N=100-x^2$. (N being the number of remaining discs).

The amount of Blue discs was $N/12$ meaning that N was a multiple of 12.

So I then collected all of the multiples

of 12: 12, 24, 36, 48, 60, 72, 84, 96. I then eliminated those that did not fit the earlier equation because there was not a square number that fitted. This left: 36, 84, 96.

I eliminated 96 because x had to be more than three for there was one complete row of orange discs and two of red discs.

This left: 36, 84. From this I deduced that x had to be 4 or 8. This means that the amount of red discs had to end in a 2 or a 4, because there are two incomplete rows of red these either have to be a length of 6 or 2.

Since the amount of red discs is half of N I halved both my possible N 's which came up with 18 and 42. This means that the amount of reds was 42 and N was 84.

This means that x is 4 and that the amount of orange discs was 6 meaning the fraction is 1/14.

The amount of blue was $N/12$ which was 7.

This means that the amount of green discs was 3 because blues + greens = 10. That means the fraction was 1/28.

The amount of white was 1 meaning the fraction was 1/84.

Finally the rest were yellow.

Red was 42. Blacks was $\frac{1}{4}$ which was 21. Blue was 7. Orange was 6.
Green was 3. White was 1.

If you take all those away from 84 you end up with 4. That is the amount of yellows. This means the fraction is $\frac{1}{21}$.